Introduction to Trigonometry

2016

Very Short Answer Type Questions [1 Mark]

Question 1.
Find the value of $\sec^2 42° - \text{cosec}^2 48°$.

Solution:

$$\sec^2 42° - \text{cosec}^2 48° = \sec^2 42° - \text{cosec}^2 (90° - 42°)$$
$$= \sec^2 42° - \sec^2 42° \qquad [\text{Using } \sec\theta = \text{cosec}(90° - \theta)]$$

Question 2.
If (1 + cos A) (1 – cos A) =3/4, find the value of sec A.

Solution:

$$(1 + \cos A)(1 - \cos A) = \frac{3}{4}$$
$$\therefore \quad 1 - \cos^2 A = \frac{3}{4}$$
$$1 - \frac{3}{4} = \cos^2 A$$
$$\frac{1}{4} = \cos^2 A \Rightarrow \sec^2 A = 4 \Rightarrow \sec A = \pm 2$$

Question 3.
If cosec θ + cot θ = x, find the value of cosec θ – cot θ

Solution:

$$\text{cosec}\,\theta + \cot\theta = x$$
As we know that $\text{cosec}^2\theta - \cot^2\theta = 1$
$$\Rightarrow \quad (\text{cosec}\,\theta - \cot\theta)(\text{cosec}\,\theta + \cot\theta) = 1$$
$$(\text{cosec}\,\theta - \cot\theta)x = 1$$
$$\Rightarrow \quad \text{cosec}\,\theta - \cot\theta = \frac{1}{x}$$

Short Answer Type Question I [2 Marks]

Question 4.
Write the values of sec 0°, sec 30°, sec 45°, sec 60° and sec 90°. What happens to sec x

when x increases from 0° to 90° ?

Solution:

Angles (θ)	0°	30°	45°	60°	90°
Sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined

The value of sec x increases and finally reaches to not defined limit as x increases from 0° to 90°.

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Short Answer Type Questions II [3 Marks]

Question 5.
Given tan A = 5/12 , find the other trigonometric ratios of the angle A.

Solution:

Given: $\tan A = \frac{5}{12}$

As $\cot A = \frac{1}{\tan A} = \frac{12}{5}$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A = 1 + \left(\frac{12}{5}\right)^2 = 1 + \frac{144}{25} = \frac{169}{25}$$

$$\operatorname{cosec}^2 A = \frac{169}{25}$$

$\Rightarrow$ $\operatorname{cosec} A = \frac{13}{5}$

Now, $\operatorname{cosec} A = \frac{1}{\sin A} \Rightarrow \frac{13}{5} = \frac{1}{\sin A}$

$$\sin A = \frac{5}{13}$$

Also, $\cos^2 A = 1 - \sin^2 A = 1 - \frac{25}{169} = \frac{169 - 25}{169} = \frac{144}{169}$

$$\cos^2 A = \frac{144}{169}$$

$\therefore$ $\cos A = \frac{12}{13}$

As $\cos A = \frac{1}{\sec A}$

$\therefore$ $\sec A = \frac{13}{12}$

Question 6.
Prove that 1/sec A – tan A-1/cosA=1/ cos A -1/sec A + tan A

Solution:

$$\text{LHS} = \frac{1}{\sec A - \tan A} - \frac{1}{\cos A} = \frac{\sec A + \tan A}{(\sec A - \tan A)(\sec A + \tan A)} - \frac{1}{\cos A}$$

(Rationalising the denominator of I[st] fraction by sec A + tan A)

$$= \frac{\sec A + \tan A}{\sec^2 A - \tan^2 A} - \sec A = \sec A + \tan A - \sec A \quad (\text{as } \sec^2 A - \tan^2 A = 1)$$

$$= \tan A$$

$$\text{RHS} = \frac{1}{\cos A} - \frac{1}{\sec A + \tan A} = \frac{1}{\cos A} - \frac{(\sec A - \tan A)}{(\sec^2 A - \tan^2 A)}$$

(Rationalising the denominator of 2nd fraction by sec A – tan A)

$$= \sec A - \frac{(\sec A - \tan A)}{\sec^2 A - \tan^2 A} = \sec A - (\sec A - \tan A)$$

$$= \sec A - \sec A + \tan A = \tan A$$

Now, LHS = RHS   Hence proved

Question 7.
If sin θ = 12/13, 0° < θ < 90°, find the value of: sin² θ- cos² θ /2 sin θ. cos θ x 1/tan² θ

Solution:

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Given: $\sin\theta = \frac{12}{13}$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$= \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{169-144}{169}} = \sqrt{\frac{25}{169}}$$

$$\cos\theta = \frac{5}{13}$$

$$\Rightarrow \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$$

Now, put values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ in the given expression

We get $$\frac{\sin^2\theta-\cos^2\theta}{2\sin\theta\cdot\cos\theta}\times\frac{1}{\tan^2\theta} = \frac{\left(\frac{12}{13}\right)^2-\left(\frac{5}{13}\right)^2}{2\times\frac{12}{13}\times\frac{5}{13}}\times\frac{1}{\left(\frac{12}{5}\right)^2}$$

$$= \frac{\frac{144}{169}-\frac{25}{169}}{\frac{120}{169}}\times\frac{25}{144} = \frac{119}{120}\times\frac{25}{144} = \frac{595}{3456}$$

Long Answer Type Questions [4 Marks]

Question 8.

If sin (A + B) = 1 and tan (A – B) = 1/√3, find the value of:

1. tan A + cot B
2. sec A – cosec B

Solution:

$\sin(A + B) = 1$ (Given)

$\Rightarrow \sin(A + B) = \sin 90^\circ$ (As $\sin 90^\circ = 1$)

$\Rightarrow A + B = 90^\circ$ ...(*i*)

Also $\tan(A - B) = \frac{1}{\sqrt{3}}$ (Given)

$\tan(A - B) = \tan 30^\circ$ (As $\tan 30^\circ = \frac{1}{\sqrt{3}}$)

$\Rightarrow A - B = 30^\circ$ ...(*ii*)

Solving (*i*) and (*ii*) for A and B, we get $A = 60^\circ$ and $B = 30^\circ$

(*i*) $\tan A + \cot B = \tan 60^\circ + \cot 30^\circ = \sqrt{3}+\sqrt{3} = 2\sqrt{3}$

(*ii*) $\sec A - \text{cosec } B = \sec 60^\circ - \text{cosec } 30^\circ = 2 - 2 = 0$

Question 9.

If sec A = x + 1/4x, prove that sec A + tan A = 2x or 1/2x

Solution:

Get More Learning Materials Here : CLICK HERE www.studentbro.in

We have $\sec A = x + \frac{1}{4x}$ (Given) ...(*i*)

we know that $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \left(x + \frac{1}{4x}\right)^2 - \tan^2 A = 1$$

$$\Rightarrow x^2 + \frac{1}{16x^2} + 2(x)\left(\frac{1}{4x}\right) - \tan^2 A = 1$$

$$x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1 = \tan^2 A$$

$$x^2 + \frac{1}{16x^2} - \frac{1}{2} = \tan^2 A$$

$$\Rightarrow \tan^2 A = x^2 + \frac{1}{16x^2} - 2(x)\left(\frac{1}{4x}\right)$$

$$\tan^2 A = \left(x - \frac{1}{4x}\right)^2$$

$$\Rightarrow \tan A = \pm\left(x - \frac{1}{4x}\right)$$

$$\tan A = \left(x - \frac{1}{4x}\right) \text{ or } \left(-x + \frac{1}{4x}\right) \quad ...(ii)$$

Adding (*i*) and (*ii*) we get

$$\sec A + \tan A = x + \frac{1}{4x} + x - \frac{1}{4x} = 2x$$

and $$\sec A + \tan A = x + \frac{1}{4x} - x + \frac{1}{4x} = \frac{1}{2x}.$$

Question 10.

Prove that: $$\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta \operatorname{cosec}\theta - 2\sin\theta\cos\theta$$

Solution:

$$\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \frac{\tan^3\theta}{\sec^2\theta} + \frac{\cot^3\theta}{\operatorname{cosec}^2\theta} = \frac{\sin^3\theta}{\cos^3\theta}.\cos^2\theta + \frac{\cos^3\theta}{\sin^3\theta}.\sin^2\theta$$

$$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta} = \frac{\sin^4\theta + \cos^4\theta}{\sin\theta\cos\theta} = \frac{(\sin^2\theta)^2 + (\cos^2\theta)^2}{\sin\theta\cos\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta} \quad [\text{Using } a^2+b^2 = (a+b)^2 - 2ab]$$

$$= \frac{1 - 2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} - \frac{2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \sec\theta \operatorname{cosec}\theta - 2\sin\theta\cos\theta$$

Question 11.

If A + B = 90°, prove that: $$\sqrt{\frac{\tan A\tan B + \tan A\cot B}{\sin A\sec B} - \frac{\sin^2 B}{\cos^2 A}} = \tan A$$

Solution:

Get More Learning Materials Here : CLICK HERE www.studentbro.in

$$A + B = 90°$$
$$\Rightarrow \quad A = 90° - B$$

So, $\sqrt{\dfrac{\tan A \tan B + \tan A \cot B}{\sin A \sec B} - \dfrac{\sin^2 B}{\cos^2 A}}$

$$= \sqrt{\frac{\tan(90° - B).\tan B + \tan(90° - B)\cot B}{\sin(90° - B)\sec B} - \frac{\sin^2 B}{\cos^2(90° - B)}}$$

$$= \sqrt{\frac{\cot B.\tan B + \cot B.\cot B}{\cos B \sec B} - \frac{\sin^2 B}{\sin^2 B}} \qquad \begin{bmatrix} \text{as } \tan(90° - \theta) = \cot\theta \\ \sin(90° - \theta) = \cos\theta \\ \cos(90° - \theta) = \sin\theta \end{bmatrix}$$

$$= \sqrt{\frac{1 + \cot^2 B}{1} - 1} = \sqrt{1 + \cot^2 B - 1} = \sqrt{\cot^2 B} = \cot B = \cot(90° - A) = \tan A$$

Question 12.

If sec θ – tan θ = x, show that: sec θ=1/2(x+1/x) and tan θ=1/2(1/x-x)

Solution:

Given: $\sec\theta - \tan\theta = x$ ...(*i*)

as we know $\sec^2\theta - \tan^2\theta = 1$

$$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$$
$$x(\sec\theta + \tan\theta) = 1$$
$$\sec\theta + \tan\theta = \frac{1}{x} \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2\sec\theta = x + \frac{1}{x} \quad \Rightarrow \quad \sec\theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$$

Subtracting (*i*) from (*ii*), we get

$$2\tan\theta = \frac{1}{x} - x$$
$$\tan\theta = \frac{1}{2}\left(\frac{1}{x} - x\right)$$

2015

Very Short Answer Type Questions [1 Mark]

Question 13.

Evaluate: sin θ.sec(9O – θ)

Solution:

$$\sin\theta \sec(90 - \theta) = \sin\theta.\operatorname{cosec}\theta = \sin\theta.\frac{1}{\sin\theta} = 1$$

Question 14.

Find the value of (cosec² θ – l).tan²θ

Solution:

$$(\operatorname{cosec}^2\theta - 1).\tan^2\theta = \cot^2\theta.\tan^2\theta = \cot^2\theta.\frac{1}{\cot^2\theta} = 1$$

Short Answer Type Question I [2 Marks]

Question 15.

Prove the following identity:sin³ θ+cos³ θ/sin θ+cos θ= 1 – sin θ.cos θ

Solution:

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Consider $\quad \text{LHS} = \dfrac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta}$

$$= \frac{(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)}{(\sin\theta + \cos\theta)}$$

$$= (\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)$$

$$= 1 - \sin\theta\cos\theta \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \text{RHS}$$

Short Answer Type Questions II [3 Marks]

Question 16.

If 7sin²A + 3cos²A = 4, show that tan A =1/√3

Solution:

Given, $7\sin^2A + 3\cos^2A = 4$

Dividing both sides by $\cos^2A$, we get

$$7\tan^2A + 3 = 4\sec^2A$$

$$\Rightarrow \quad 7\tan^2A + 3 = 4(1 + \tan^2A) \qquad [\because \sec^2\theta = 1 + \tan^2\theta]$$

$$7\tan^2A + 3 = 4 + 4\tan^2A$$

$$\Rightarrow \quad 3\tan^2A = 1$$

$$\Rightarrow \quad \tan^2A = \frac{1}{3} \Rightarrow \tan A = \frac{1}{\sqrt{3}}$$

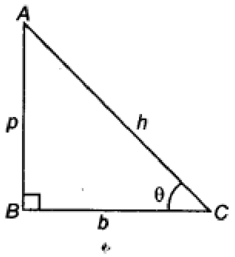
Hence proved.

Question 17.

For any acute angle θ, prove that

1. sin²θ + cos²θ = 1
2. 1 + cot²θ = cosec²θ

Solution:



(i) $\quad \text{LHS} = \sin^2\theta + \cos^2\theta$

$$= \left(\frac{p}{h}\right)^2 + \left(\frac{b}{h}\right)^2$$

$$= \frac{p^2 + b^2}{h^2} = \frac{h^2}{h^2} = 1 = \text{RHS}$$

(ii) $\quad \text{LHS} = 1 + \cot^2\theta$

$$= 1 + \left(\frac{b}{p}\right)^2 = 1 + \frac{b^2}{p^2}$$

$$= \frac{p^2 + b^2}{p^2} = \frac{h^2}{p^2} = \left(\frac{h}{p}\right)^2 = \text{cosec}^2\theta = \text{RHS}$$

Long Answer Type Questions[4 Marks]

Question 18.

Prove that: $\sqrt{\sec^2\theta + \text{cosec}^2\theta} = \tan\theta + \cot\theta$

Solution:

Consider, $\quad \text{LHS} = \sqrt{\sec^2\theta + \text{cosec}^2\theta}$

$$= \sqrt{1 + \tan^2\theta + 1 + \cot^2\theta} = \sqrt{\tan^2\theta + \cot^2\theta + 2}$$

$$= \sqrt{(\tan\theta + \cot\theta)^2} = \tan\theta + \cot\theta = \text{RHS.}$$

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Question 19.

Prove that:

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{1 - 2\cos^2 A}$$

Solution:

$$\text{LHS} = \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$$

$$= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}$$

$$= \frac{\sin^2 A + \cos^2 A + 2\sin A\cos A + \sin^2 A + \cos^2 A - 2\sin A\cos A}{\sin^2 A - \cos^2 A}$$

$$= \frac{2(\sin^2 A + \cos^2 A)}{1 - \cos^2 A - \cos^2 A} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2}{1 - 2\cos^2 A} = \text{RHS}$$

2014

Very Short Answer Type Questions [1 Mark]

Question 20.

If sinθ = x and sec θ = y then find the value of cot θ.

Solution:

Given $\sin\theta = x$ and $\sec\theta = y \Rightarrow \cos\theta = \frac{1}{y}$

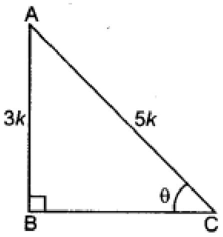
Now, $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{xy}$

Hence, $\cot\theta = \frac{1}{xy}$

Question 21.

If cosec θ = 5/3, then what is the value of cos θ + tanθ

Solution:



$\because \quad \text{cosec}\,\theta = \frac{5}{3}$

In right angled ΔABC, ∠B = 90°

So, $AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow (5k)^2 = (3k)^2 + BC^2$

$\Rightarrow BC^2 = 25k^2 - 9k^2$

$\Rightarrow BC^2 = 16k^2 \Rightarrow BC = 4k$

So, $\cos\theta = \frac{4}{5}$ and $\tan\theta = \frac{3}{4}$

Now, $\cos\theta + \tan\theta = \frac{4}{5} + \frac{3}{4} = \frac{16+15}{20} = \frac{31}{20}$

Question 22.

Find the value of tan(65° – θ) – cot(25° + θ)

Solution:

$\tan(65° - \theta) - \cot(25° + \theta)$

$= \cot\{90° - (65° - \theta)\} - \cot(25° + \theta)$

$= \cot(25° + \theta) - \cot(25° + \theta) = 0$

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Question 23.
Find the value of sin 38° – cos 52°.

Solution:

$$\sin 38^\circ - \cos 52^\circ = \cos(90^\circ - 38^\circ) - \cos 52^\circ = \cos 52^\circ - \cos 52^\circ = 0$$

Question 24.
Find the value of cos θ + sec θ, when it is given that cos θ =1/2

Solution:

$$\because \quad \cos\theta = \frac{1}{2} \Rightarrow \sec\theta = 2$$

$$\text{Now,} \quad \cos\theta + \sec\theta = \frac{1}{2} + 2 = \frac{1+4}{2} = \frac{5}{2}$$

Question 25.
Evaluate: 3 cot² 60° + sec² 45°.

Solution:

$$3\cot^2 60^\circ + \sec^2 45^\circ = 3\left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{2})^2 = 3 \times \frac{1}{3} + 2 = 1 + 2 = 3$$

Short Answer Type Questions I [2 Marks]

Question 26.
Solve the equation for θ: cos²θ/cot²θ – cos² θ=3

Solution:

$$\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3 \Rightarrow \frac{\cos^2\theta}{\cos^2\theta\left(\frac{1}{\sin^2\theta} - 1\right)} = 3$$

$$\Rightarrow \frac{1}{\operatorname{cosec}^2\theta - 1} = 3 \Rightarrow \frac{1}{\cot^2\theta} = 3$$

$$\Rightarrow \tan^2\theta = 3 \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$$

$$\text{Hence,} \quad \theta = 60^\circ$$

Question 27.
Express cos A in terms of cot A.

Solution:

$$\because \quad \sec^2 A = 1 + \tan^2 A$$

$$\Rightarrow \frac{1}{\cos^2 A} = 1 + \frac{1}{\cot^2 A} \Rightarrow \frac{1}{\cos^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \cos^2 A = \frac{\cot^2 A}{1 + \cot^2 A} \Rightarrow \cos A = \frac{\cot A}{\sqrt{1 + \cot^2 A}}$$

Question 28.
If A, B, and C are the interior angles of a ΔABC, show that tan(A+B/2)=cot C/2

Solution:

$$\text{In any } \Delta ABC, \quad \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + \angle B = 180^\circ - \angle C$$

$$\Rightarrow \frac{\angle A + \angle B}{2} = 90^\circ - \frac{\angle C}{2}$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \tan\left(90^\circ - \frac{C}{2}\right)$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \cot\frac{C}{2}$$

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Question 29.

If sin A = cos A, find the value of $2\tan^2 A + \sin^2 A + 1$.

Solution:

$\because$ $\sin A = \cos A$

$\Rightarrow$ $A = 45°$

Now, $2\tan^2 A + \sin^2 A + 1 = 2\tan^2 45° + \sin^2 45° + 1$

$$= 2 \times (1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + 1$$

$$= 2 + \frac{1}{2} + 1 = \frac{7}{2}.$$

Question 30.

If tan θ + cot θ= 2, find the value of $\sqrt{\tan^2\theta + \cot^2\theta}$.

Solution:

Given $\tan\theta + \cot\theta = 2$

Squaring both sides, we get

$$\tan^2\theta + \cot^2\theta + 2 = 4$$

$\Rightarrow$ $\tan^2\theta + \cot^2\theta = 2$

Taking square root on both sides, we get

$$\sqrt{\tan^2\theta + \cot^2\theta} = \sqrt{2}$$

Question 31.

If tan(A – B) = 1/√3 and tan (A + B) = √3, find A and B.

Solution:

$\because$ $\tan(A - B) = \frac{1}{\sqrt{3}}$ $\Rightarrow$ $A - B = 30°$ ...(*i*)

and $\tan(A + B) = \sqrt{3}$ $A + B = 60°$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$$A - B = 30°$$
$$A + B = 60°$$
$$2A = 90° \Rightarrow A = 45°$$

Putting A = 45° in (*i*), we get

$$45° - B = 30°$$

$\Rightarrow$ $B = 15°$

Hence, A = 45° and B = 15°.

Question 32.

If ac = r cos θ. sin Φ; y = r sinθ. sinΦ; z = r cos Φ. Prove that $x^2 + y^2 + z^2 = r^2$.

Solution:

$\because$ $x = r\cos\theta\sin\phi$ $y = r\sin\theta\sin\phi$ $z = r\cos\phi$

$\Rightarrow$ $x^2 = r^2\cos^2\theta\sin^2\phi$ ...(*i*) $y^2 = r^2\sin^2\theta\sin^2\phi$ ...(*ii*) $z^2 = r^2\cos^2\phi$ ...(*iii*)

Adding (*i*), (*ii*) and (*iii*), we get

$$x^2 + y^2 + z^2 = r^2\cos^2\theta\sin^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\phi$$
$$= r^2\sin^2\phi(\cos^2\theta + \sin^2\theta) + r^2\cos^2\phi$$
$$= r^2\sin^2\phi.1 + r^2\cos^2\phi$$
$$= r^2(\sin^2\phi + \cos^2\phi) = r^2.1 = r^2$$

Question 33.

Evaluate: $\dfrac{3\tan 25° \cdot \tan 40° \cdot \tan 50° \cdot \tan 65° - \frac{1}{2}\tan^2 60°}{4(\cos^2 29° + \cos^2 61°)}$

Solution:

Get More Learning Materials Here : CLICK HERE www.studentbro.in

$$\frac{3\tan 25^\circ \tan 40^\circ \tan 50^\circ \tan 65^\circ - \frac{1}{2}\tan^2 60^\circ}{4(\cos^2 29^\circ + \cos^2 61^\circ)}$$

$$= \frac{3\tan 25^\circ \tan 40^\circ \tan(90^\circ - 40^\circ)\tan(90^\circ - 25^\circ) - \frac{1}{2}\times(\sqrt{3})^2}{4\{\cos^2 29^\circ + \cos^2(90^\circ - 29^\circ\}}$$

$$= \frac{3\tan 25^\circ \tan 40^\circ \cot 40^\circ \cot 25^\circ - \frac{(\sqrt{3})^2}{2}}{4(\cos^2 29^\circ + \sin^2 29^\circ)} = \frac{3 - \frac{3}{2}}{4} = \frac{6-3}{8} = \frac{3}{8}$$

Question 34.

Prove that $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

Solution:

$$\text{LHS} = \frac{\cot A - \cos A}{\cot A + \cos A}$$

$$= \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A} = \frac{\cos A\left(\frac{1}{\sin A} - 1\right)}{\cos A\left(\frac{1}{\sin A} + 1\right)} = \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} = \text{RHS}$$

Long Answer Type Questions [4 Marks]

Question 35.

Given that cos(A – B) = cos A.cos B + sinA.sinB, find the value of cos 15° in two ways.

1. Taking A = 60°, B = 45° and
2. Taking A = 45°, B = 30°

Solution:

(*i*) By taking A = 60° and B = 45°

$$\cos 15^\circ = \cos(60^\circ - 45^\circ)$$
$$= \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ$$
$$= \frac{1}{2}\times\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}$$

(*ii*) By taking A = 45° and B = 30°

$$\cos 15^\circ = \cos(45^\circ - 30^\circ)$$
$$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$
$$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\times\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Question 36.

If cosec A + cot A = m, show that m^2-1/ m^2 + 1= cos A.

Solution:

Get More Learning Materials Here : CLICK HERE www.studentbro.in

$$\text{LHS} = \frac{m^2-1}{m^2+1} \qquad [\because \text{cosec A} + \cot A = m]$$

$$= \frac{(\text{cosec A}+\cot A)^2-1}{(\text{cosec A}+\cot A)^2+1} = \frac{\text{cosec}^2 A+\cot^2 A+2\,\text{cosec A}\cot A-1}{\text{cosec}^2 A+\cot^2 A+2\,\text{cosec A}\cot A+1}$$

$$= \frac{(\text{cosec}^2 A-1)+\cot^2 A+2\,\text{cosec A}\cot A}{\text{cosec}^2 A+(1+\cot^2 A)+2\,\text{cosec A}\cot A}$$

$$= \frac{\cot^2 A+\cot^2 A+2\,\text{cosec A}\cot A}{\text{cosec}^2 A+\text{cosec}^2 A+2\,\text{cosec A}\cot A} = \frac{2\cot^2 A+2\,\text{cosec A}\cot A}{2\,\text{cosec}^2 A+2\,\text{cosec A}\cot A}$$

$$= \frac{2\cot A(\cot A+\text{cosec A})}{2\,\text{cosec A}(\text{cosec A}+\cot A)} = \frac{\cot A}{\text{cosec A}} = \frac{\cos A}{\sin A\,\text{cosec A}} = \cos A = \text{RHS}$$

Hence, $\cos A = \dfrac{m^2-1}{m^2+1}$.

Question 37.

Prove that: (sec θ + tan θ)² = cosec θ +1/cosec θ -1

Solution:

Consider $\text{LHS} = (\sec\theta + \tan\theta)^2$

$$= \left(\frac{1+\sin\theta}{\cos\theta}\right)^2 = \frac{(1+\sin\theta)^2}{\cos^2\theta} = \frac{(1+\sin\theta)^2}{1-\sin^2\theta} = \frac{1+\sin\theta}{1-\sin\theta}$$

$$= \frac{\left(\dfrac{1+\sin\theta}{\sin\theta}\right)}{\left(\dfrac{1-\sin\theta}{\sin\theta}\right)} = \frac{\left(\dfrac{1}{\sin\theta}+1\right)}{\left(\dfrac{1}{\sin\theta}-1\right)} = \frac{\text{cosec}\,\theta+1}{\text{cosec}\,\theta-1} = \text{RHS}$$

Question 38.

If cosec θ + cot θ = q, show that cosec θ – cot θ = 1/q and hence find the values of sin θ and sec θ

Solution:

Given $\text{cosec}\,\theta + \cot\theta = q$ ...(*i*)

Now, $\text{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow (\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta) = 1$

$\Rightarrow q.(\text{cosec}\,\theta - \cot\theta) = 1$

$\Rightarrow \text{cosec}\,\theta - \cot\theta = \dfrac{1}{q}$ ...(*ii*)

On adding (*i*) and (*ii*), we get

$$2\text{cosec}\,\theta = q + \frac{1}{q} \Rightarrow \text{cosec}\,\theta = \frac{q^2+1}{2q} \Rightarrow \sin\theta = \frac{2q}{q^2+1}$$

On subtracting (*ii*) from (*i*), we get

$$2\cot\theta = q - \frac{1}{q} \Rightarrow \cot\theta = \frac{q^2-1}{2q} \Rightarrow \frac{\cos\theta}{\sin\theta} = \frac{q^2-1}{2q}$$

$$\Rightarrow \cos\theta = \frac{(q^2-1)}{2q}\times\sin\theta \Rightarrow \cos\theta = \frac{(q^2-1)}{2q}\times\frac{2q}{(q^2+1)} = \frac{q^2-1}{q^2+1}$$

$$\sec\theta = \frac{q^2+1}{q^2-1}$$

Hence, $\sin\theta = \dfrac{2q}{q^2+1}$ and $\sec\theta = \dfrac{q^2+1}{q^2-1}$

Question 39.

Prove that $\dfrac{\tan A}{\sec A - 1} + \dfrac{\tan A}{\sec A + 1} = 2\,\text{cosec A}.$

Solution:

Get More Learning Materials Here : CLICK HERE www.studentbro.in

$$\text{LHS} = \frac{\tan A}{\sec A - 1} + \frac{\tan A}{\sec A + 1} = \tan A\left(\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1}\right)$$

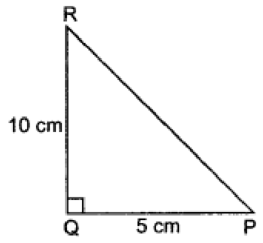
$$= \tan A\left(\frac{\sec A + 1 + \sec A - 1}{\sec^2 A - 1}\right) = \frac{\tan A(2\sec A)}{\tan^2 A} = \frac{2\sec A}{\tan A}$$

$$= \frac{\frac{2}{\cos A}}{\frac{\sin A}{\cos A}} = \frac{2}{\sin A} = 2\,\text{cosec}\,A = \text{RHS}$$

Question 40.

ΔRPQ is a right angled at Q. If PQ = 5 cm and RQ = 10 cm, find:

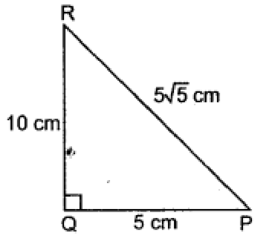
1. $\sin^2 P$
2. $\cos^2 R$ and tan R
3. sin P x cos P
4. $\sin^2 P - \cos^2 P$



Solution:

In right angled ΔRPQ, $\angle Q = 90°$

So,
$$PR^2 = 10^2 + 5^2$$
$$PR^2 = 125$$
$$PR = 5\sqrt{5}\text{ cm}$$



(*i*) $$\sin^2 P = \left(\frac{10}{5\sqrt{5}}\right)^2 = \frac{4}{5}$$

(*ii*) $$\cos^2 R = \left(\frac{10}{5\sqrt{5}}\right)^2 = \frac{4}{5} \text{ and } \tan R = \frac{5}{10} = \frac{1}{2}$$

(*iii*) $$\sin P \times \cos P = \frac{10}{5\sqrt{5}} \times \frac{5}{5\sqrt{5}} = \frac{2}{5}$$

(*iv*) $$\sin^2 P - \cos^2 P = \left(\frac{10}{5\sqrt{5}}\right)^2 - \left(\frac{5}{5\sqrt{5}}\right)^2 = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

2013

Short Answer Type Question I [2 Marks]

Question 41.

If cos θ + sin θ =√2 cos θ, show that cos θ – sin θ = √2 sin θ.

Solution:

Here,
$$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$$
$$\sin\theta = \sqrt{2}\cos\theta - \cos\theta$$
$$\sin\theta = (\sqrt{2} - 1)\cos\theta$$
$$(\sqrt{2} + 1)\sin\theta = (\sqrt{2} - 1)(\sqrt{2} + 1)\cos\theta$$
$$\sqrt{2}\sin\theta + \sin\theta = \cos\theta$$
$$\sqrt{2}\sin\theta = \cos\theta - \sin\theta$$

Hence proved

Short Answer Type Questions II [3 Marks]

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Question 42.

If sin A = cos A, find the value of $2\tan^2 A + \sin^2 A - 1$

Solution:

$$\because \quad \sin A = \cos A$$
$$\Rightarrow \quad A = 45^\circ$$
Now, $$2\tan^2 A + \sin^2 A - 1 = 2\tan^2 45^\circ + \sin^2 45^\circ - 1$$
$$= 2(1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 2 + \frac{1}{2} - 1 = \frac{3}{2}$$

Question 43.

Show that $\text{cosec}^2 \theta - \tan^2(90^\circ - \theta) = \sin^2 \theta + \sin^2(90^\circ - \theta)$.

Solution:

$$\text{LHS} = \text{cosec}^2\theta - \tan^2(90^\circ - \theta) = \text{cosec}^2\theta - \cot^2\theta = 1$$
$$\text{RHS} = \sin^2\theta + \sin^2(90^\circ - \theta) = \sin^2\theta + \cos^2\theta = 1$$

Hence, LHS = RHS.

Question 44.

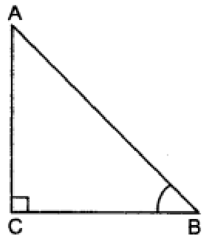
ABC is a triangle right angled at C and AC = √3 BC. Prove that ∠ABC=60°

Solution:

Here, in ΔABC, ∠C = 90° and $AC = \sqrt{3}BC$ [given]

$$\Rightarrow \quad \frac{AC}{BC} = \sqrt{3} \quad \text{...(i)}$$

Also, $$\tan B = \frac{AC}{BC}$$
$$\Rightarrow \quad \tan B = \sqrt{3} \quad \text{[Using (i)]}$$
$$\Rightarrow \quad B = 60^\circ$$



Question 45.

Show that $\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} = \text{cosec}\,\alpha + \cot\alpha$.

Solution:

$$\text{LHS} = \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} = \left(\frac{\sqrt{1+\cos\alpha}}{\sqrt{1-\cos\alpha}}\right)\times\left(\frac{\sqrt{1+\cos\alpha}}{\sqrt{1+\cos\alpha}}\right) = \frac{1+\cos\alpha}{\sqrt{1-\cos^2\alpha}}$$
$$= \frac{1+\cos\alpha}{\sqrt{\sin^2\alpha}} = \frac{1+\cos\alpha}{\sin\alpha} = \frac{1}{\sin\alpha} + \frac{\cos\alpha}{\sin\alpha} = \text{cosec}\,\alpha + \cot\alpha = \text{RHS}$$

Question 46.

If tan(A – B) = 1/√3 and tan (A + B) = √3, find A and B

Solution:

Refer to sol of Question no. 19.

Long Answer Type Questions [4 Marks]

Question 47.

Evaluate: $\frac{4\cot^2 60^\circ + \sec^2 30^\circ - 2\sin^2 45^\circ}{\sin^2 60^\circ + \cos^2 45^\circ}$

Solution:

Get More Learning Materials Here : CLICK HERE www.studentbro.in

$$\frac{4\cot^2 60° + \sec^2 30° - 2\sin^2 45°}{\sin^2 60° + \cos^2 45°} = \frac{4\times\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 - 2\times\left(\frac{1}{\sqrt{2}}\right)^2}{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \frac{\frac{4}{3} + \frac{4}{3} - 2\times\frac{1}{2}}{\frac{3}{4} + \frac{1}{2}} = \frac{\frac{8}{3} - 1}{\frac{3+2}{4}} = \frac{\frac{5}{3}}{\frac{5}{4}} = \frac{5}{3}\times\frac{4}{5} = \frac{4}{3}$$

Question 48.

If sec θ + tan θ = p, then find the value of cosecθ

Solution:

Here, $\sec\theta + \tan\theta = p$ ...(*i*)

Now, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow p.(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$$2\sec\theta = p + \frac{1}{p} \Rightarrow \sec\theta = \frac{p^2+1}{2p}$$

On subtracting (*ii*) from (*i*), we get

$$2\tan\theta = p - \frac{1}{p} \Rightarrow \tan\theta = \frac{p^2-1}{2p}$$

Now, $$\frac{\sec\theta}{\tan\theta} = \frac{\frac{p^2+1}{2p}}{\frac{p^2-1}{2p}} \Rightarrow \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \frac{p^2+1}{p^2-1}$$

$$\Rightarrow \frac{1}{\sin\theta} = \frac{p^2+1}{p^2-1} \Rightarrow \text{cosec}\,\theta = \frac{p^2+1}{p^2-1}$$

Question 49.

Evaluate: 4/Cot² 30°+1/sin² 60°-cos² 45°

Solution:

Here, $$\frac{4}{\cot^2 30°} + \frac{1}{\sin^2 60°} - \cos^2 45° = \frac{4}{(\sqrt{3})^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{4}{3} + \frac{4}{3} - \frac{1}{2} = \frac{8+8-3}{6} = \frac{13}{6}$$

Question 50.

Evaluate: 4(sin430° + cos460°) – 3(cos²45°-sin²90°)

Solution:

$4(\sin^4 30° + \cos^4 60°) - 3(\cos^2 45° - \sin^2 90°)$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - 1^2\right] = 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right]$$

$$= 4\times\frac{2}{16} - 3\left(-\frac{1}{2}\right) = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$

Question 51.

Prove that 1/cosecA+cotA-1/sinA=1/sinA-1/cosecA-cotA

Solution:

Get More Learning Materials Here : CLICK HERE www.studentbro.in

We have to prove

$$\frac{1}{\operatorname{cosec} A + \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A - \cot A}$$

or $$\frac{1}{\operatorname{cosec} A + \cot A} + \frac{1}{\operatorname{cosec} A - \cot A} = \frac{2}{\sin A}$$

Now,

$$\text{LHS} = \frac{1}{\operatorname{cosec} A + \cot A} + \frac{1}{\operatorname{cosec} A - \cot A}$$

$$= \frac{\operatorname{cosec} A - \cot A + \operatorname{cosec} A + \cot A}{\operatorname{cosec}^2 A - \cot^2 A}$$

$$= \frac{2 \operatorname{cosec} A}{1} = \frac{2}{\sin A} = \text{RHS.}$$

2012

Short Answer Type Questions I [2 Marks]

Question 52.

If √3 sin θ – cos θ = 0 and 0° < θ < 90°, find the value of θ.

Solution:

$$\sqrt{3} \sin \theta - \cos \theta = 0$$

$$\Rightarrow \quad \sqrt{3} \sin \theta = \cos \theta \Rightarrow \sqrt{3} = \frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \quad \cot \theta = \sqrt{3} \Rightarrow \theta = 30^\circ$$

Question 53.

If sin A = √3/2, find the value of 2 cot² A -1.

Solution:

$$\because \quad \sin A = \frac{\sqrt{3}}{2} \therefore A = 60^\circ$$

Now $$2 \cot^2 A - 1 = 2.\cot^2 60^\circ - 1 = 2 \times \left(\frac{1}{\sqrt{3}}\right)^2 - 1 = \frac{2}{3} - 1 = -\frac{1}{3}$$

Short Answer Type Questions II [3 Marks]

Question 54.

Find the value of x if $$4\left(\frac{\sec^2 59^\circ - \cot^2 31^\circ}{3}\right) - \frac{2}{3} \sin 90^\circ + 3 \tan^2 56^\circ \times \tan^2 34^\circ = \frac{x}{3}$$

Solution:

$$4\left(\frac{\sec^2 59^\circ - \cot^2 31^\circ}{3}\right) - \frac{2}{3} \sin 90^\circ + 3 \tan^2 56^\circ \tan^2 34^\circ = \frac{x}{3}$$

$$\Rightarrow \quad 4\left(\frac{\sec^2 59^\circ - \tan^2(90^\circ - 31^\circ)}{3}\right) - \frac{2}{3} \times 1 + 3 \tan^2 56^\circ \cdot \cot^2 (90^\circ - 34^\circ) = \frac{x}{3}$$

$$\Rightarrow \quad 4\left(\frac{\sec^2 59^\circ - \tan^2 59^\circ}{3}\right) - \frac{2}{3} + 3 \tan^2 56^\circ \cot^2 56^\circ = \frac{x}{3}$$

$$\Rightarrow \quad 4 \times \frac{1}{3} - \frac{2}{3} + 3 \times 1 = \frac{x}{3} \Rightarrow \frac{4}{3} - \frac{2}{3} + 3 = \frac{x}{3} \Rightarrow x = 11$$

Long Answer Type Questions [4 Marks]

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Question 55.

If A + B = 90°, prove that

$$\sqrt{\frac{\tan A \tan B + \tan A \cot B}{\sin A \sec B} - \frac{\sin^2 B}{\cos^2 A}} = \tan A$$

Solution:

$A + B = 90° \Rightarrow B = 90° - A$

Now; $\text{LHS} = \sqrt{\frac{\tan A.\tan B + \tan A.\cot B}{\sin A.\sec B} - \frac{\sin^2 B}{\cos^2 A}}$

$$= \sqrt{\frac{\tan A.\tan(90° - A) + \tan A.\cot(90° - A)}{\sin A.\sec(90° - A)} - \frac{\sin^2(90° - A)}{\cos^2 A}}$$

$$= \sqrt{\frac{\tan A.\cot A + \tan A.\tan A}{\sin A.\text{cosec} A} - \frac{\cos^2 A}{\cos^2 A}}$$

$$= \sqrt{\frac{1 + \tan^2 A}{1} - 1} = \sqrt{\tan^2 A} = \tan A = \text{RHS}$$

Question 56.

In an acute angled triangle ABC, if sin (A + B – C) = 1/2 and cos (B + C – A) = 1/√2 find ∠A, ∠B and ∠C

Solution:

$\because \quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle A + \angle B = 180° - \angle C \quad \ldots(i)$

and $\quad \angle B + \angle C = 180° - \angle A \quad \ldots(ii)$

Now, $\sin(A + B - C) = \frac{1}{2} \Rightarrow A + B - C = 30°$

$\Rightarrow \quad 180° - C - C = 30° \Rightarrow 2C = 150° \quad$ [From (i)]

$\Rightarrow \quad C = 75°$

Also $\cos(B + C - A) = \frac{1}{\sqrt{2}} \Rightarrow B + C - A = 45°$

$\Rightarrow \quad 180° - A - A = 45° \Rightarrow 2A = 135° \quad$ [From (ii)]

$\Rightarrow \quad A = 67.5°$

$\therefore \quad \angle B = 37.5°$

Hence, $\angle A = 67.5°, \angle B = 37.5°, \angle C = 75°.$

2011

Short Answer Type Questions I [2 Marks]

Question 57.

Prove that: cosA/1+sinA+1+sinA/cosA=2 secA

Solution:

Taking $\text{LHS} = \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$

$$= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A)\cos A} = \frac{\cos^2 A + \sin^2 A + 1 + 2\sin A}{(1 + \sin A)\cos A}$$

$$= \frac{2(1 + \sin A)}{(1 + \sin A)\cos A} = \frac{2}{\cos A} = 2\sec A = \text{RHS}$$

Question 58.

Evaluate :

$$\frac{\tan^2 60° + 4\sin^2 45° + 3\sec^2 30° + 5\cos^2 90°}{\text{cosec } 30° + \sec 60° - \cot^2 30°}$$

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Solution:

$$\frac{\tan^2 60° + 4\sin^2 45° + 3\sec^2 30° + 5\cos^2 90°}{\text{cosec } 30° + \sec 60° - \cot^2 30°}$$

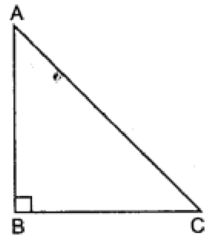
$$= \frac{(\sqrt{3})^2 + 4\times\left(\frac{1}{\sqrt{2}}\right)^2 + 3\times\left(\frac{2}{\sqrt{3}}\right)^2 + 5\times 0}{2+2-(\sqrt{3})^2} = \frac{3+2+4}{1} = 9$$

Short Answer Type Questions II [3 Marks]

Question 59.

In the figure below, ΔABC is right angled at B, BC = 7 cm and AC – AB = 1 cm. Find the value of cos A + sin A

Solution:



Let AC = x cm, AB = y cm,

$\therefore \quad x - y = 1$ ...(*i*)

Now, in right angled ΔABC, ∠B = 90°

So, $AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow x^2 = y^2 + BC^2$

$\Rightarrow x^2 - y^2 = (7)^2$

$\Rightarrow (x-y)(x+y) = 49$

$\Rightarrow 1 \times (x+y) = 49$ [Using (*i*)]

$\Rightarrow x + y = 49$ ...(*ii*)

Solving (*i*) and (*ii*), we get

$x = 25$ and $y = 24$

Now, $\cos A + \sin A = \frac{AB}{AC} + \frac{BC}{AC} = \frac{24}{25} + \frac{7}{25} = \frac{31}{25}$

Question 60.

If cosθ-sinθ=√2 sinθ,prove that cosθ+sinθ=√2 cosθ

Solution:

Here, $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

$\Rightarrow \cos\theta = \sqrt{2}\sin\theta + \sin\theta \Rightarrow \cos\theta = (\sqrt{2}+1)\sin\theta$

$\Rightarrow \frac{\cos\theta}{\sqrt{2}+1} = \sin\theta \Rightarrow \frac{\cos\theta(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} = \sin\theta$

$\Rightarrow \sqrt{2}\cos\theta - \cos\theta = \sin\theta \Rightarrow \sqrt{2}\cos\theta = \sin\theta + \cos\theta$ Hence proved.

Question 61.

If cosec (A-B) = 2, cot (A + B) = —0° < (A + B) <90°, A > B, then find A and B

Solution:

cosec (A – B) = 2

Also, cosec 30° = 2 ⇒ A – B = 30° ...(*i*)

and $\cot(A+B) = \frac{1}{\sqrt{3}}$

Also, $\cot 60° = \frac{1}{\sqrt{3}} \Rightarrow A + B = 60°$ ...(*ii*)

Solving (*i*) and (*ii*), we get A = 45°, B = 15°

Long Answer Type Questions [4 Marks]

Question 62.

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Prove that $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

Solution:

$$\text{Taking LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \frac{(\sqrt{1+\sin\theta})^2+(\sqrt{1-\sin\theta})^2}{\sqrt{1-\sin\theta}\times\sqrt{1+\sin\theta}} = \frac{1+\sin\theta+1-\sin\theta}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{2}{\sqrt{\cos^2\theta}} = \frac{2}{\cos\theta} = 2\sec\theta = \text{RHS}$$

Question 63.

Evaluate:

$$\frac{\sec^2(90^\circ-\theta)-\cot^2\theta}{2(\sin^2 25^\circ+\sin^2 65^\circ)}+\frac{2\sin^2 30^\circ\tan^2 32^\circ.\tan^2 58^\circ}{3(\sec^2 33^\circ-\cot^2 57^\circ)}$$

Solution:

$$\text{Now,}\quad \text{LHS} = \frac{\sec^2(90^\circ-\theta)-\cot^2\theta}{2(\sin^2 25^\circ+\sin^2 65^\circ)}+\frac{2\sin^2 30^\circ\cdot\tan^2 32^\circ\cdot\tan^2 58^\circ}{3(\sec^2 33^\circ-\cot^2 57^\circ)}$$

$$= \frac{\operatorname{cosec}^2\theta-\cot^2\theta}{2\{\sin^2 25^\circ+\cos^2(90^\circ-65^\circ)\}}+\frac{2\times\left(\frac{1}{2}\right)^2\times\tan^2 32^\circ\times\cot^2(90^\circ-58^\circ)}{3\{\sec^2 33^\circ-\tan^2(90^\circ-57^\circ)\}}$$

$$= \frac{1}{2(\sin^2 25^\circ+\cos^2 25^\circ)}+\frac{\frac{1}{2}\times\tan^2 32^\circ\times\cot^2 32^\circ}{3(\sec^2 33^\circ-\tan^2 33^\circ)}$$

$$= \frac{1}{2}+\frac{\frac{1}{2}\times 1}{3\times 1} = \frac{1}{2}+\frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

Question 64.

Determine the value of x such that 2 cosec² 30° + x sin² 60° -3/4 tan² 30° = 10.

Solution:

$$2\operatorname{cosec}^2 30^\circ + x\sin^2 60^\circ - \frac{3}{4}\tan^2 30^\circ = 10$$

$$\Rightarrow\quad 2\times(2)^2 + x\times\left(\frac{\sqrt{3}}{2}\right)^2-\frac{3}{4}\times\left(\frac{1}{\sqrt{3}}\right)^2 = 10$$

$$\Rightarrow\quad 8+x\times\frac{3}{4}-\frac{1}{4} = 10$$

$$\Rightarrow\quad \frac{3x}{4} = 10-8+\frac{1}{4} \Rightarrow \frac{3x}{4} = \frac{9}{4}$$

$$\Rightarrow\quad x = \frac{9}{4}\times\frac{4}{3} \Rightarrow x = 3$$

2010

Very Short Answer Type Questions [1 Mark]

Question 65.

If 3x= cosec θ and 3/x= cot θ, find the value of $3(x^2-1/x^2)$

Solution:

Get More Learning Materials Here : CLICK HERE www.studentbro.in

$$3x = \text{cosec}\,\theta \text{ and } \frac{3}{x} = \cot\theta$$

$$x = \frac{\text{cosec}\,\theta}{3} \text{ and } \frac{1}{x} = \frac{\cot\theta}{3}$$

$$3\left[x^2 - \frac{1}{x^2}\right] = 3\left[\left(\frac{\text{cosec}\,\theta}{3}\right)^2 - \left(\frac{\cot\theta}{3}\right)^2\right] = 3\left[\frac{\text{cosec}^2\theta - \cot^2\theta}{9}\right] = 3 \times \frac{1}{9} = \frac{1}{3}$$

Question 66.

If 2r = sec A and 2/x = tan A, find the value of $2(x^2-1/x^2)$

Solution:

Given, $2x = \sec A \text{ and } \frac{2}{x} = \tan A$

$$\Rightarrow \quad x = \frac{\sec A}{2} \text{ and } \frac{1}{x} = \frac{\tan A}{2}$$

Now, $2\left(x^2 - \frac{1}{x^2}\right) = 2\left[\frac{\sec^2 A}{4} - \frac{\tan^2 A}{4}\right] = 2\left[\frac{\sec^2 A - \tan^2 A}{4}\right] = 2 \times \frac{1}{4} = \frac{1}{2}$

Question 67.

If cosecθ = 2x and cot θ= 2/x, find the value of $2(x^2-1/x^2)$

Solution:

$$\text{cosec}\,\theta = 2x \text{ and } \cot\theta = \frac{2}{x}$$

$$\Rightarrow \quad x = \frac{\text{cosec}\,\theta}{2} \text{ and } \frac{1}{x} = \frac{\cot\theta}{2}$$

Now, $2\left(x^2 - \frac{1}{x^2}\right) = 2\left(\frac{\text{cosec}^2\theta}{4} - \frac{\cot^2\theta}{4}\right) = 2\left(\frac{\text{cosec}^2\theta - \cot^2\theta}{4}\right) = 2\left(\frac{1}{4}\right) = \frac{1}{2}$

Question 68.

If 5x = secθ and 5/x = tanθ, find the value of $5(x^2-1/x^2)$

Solution:

$$5x = \sec\theta \text{ and } \frac{5}{x} = \tan\theta$$

$$x = \frac{\sec\theta}{5} \text{ and } \frac{1}{x} = \frac{\tan\theta}{5}$$

Now, $5\left(x^2 - \frac{1}{x^2}\right) = 5\left[\left(\frac{\sec\theta}{5}\right)^2 - \left(\frac{\tan\theta}{5}\right)^2\right] = 5\left(\frac{\sec^2\theta - \tan^2\theta}{25}\right) = 5 \times \frac{1}{25} = \frac{1}{5}$

Question 69.

If 7x = cosecθ and 7/x = cot θ, find the value of (x^2-1/x^2)

Solution:

Given $7x = \text{cosec}\,\theta \text{ and } \frac{7}{x} = \cot\theta$ ...(*i*)

Consider $x^2 - \frac{1}{x^2} = \frac{1}{49}\left(49x^2 - \frac{49}{x^2}\right) = \frac{1}{49}\left\{(7x)^2 - \left(\frac{7}{x}\right)^2\right\}$

$= \frac{1}{49}(\text{cosec}^2\theta - \cot^2\theta)$ [From (*i*)]

$= \frac{1}{49} \times 1 = \frac{1}{49}$

Question 70.

If 6x = sec θ and 6/x = tanθ, find the value of $9(x^2-1/x^2)$

Solution:

Get More Learning Materials Here : CLICK HERE www.studentbro.in

$$6x = \sec\theta \text{ and } \frac{6}{x} = \tan\theta \Rightarrow x = \frac{\sec\theta}{6} \text{ and } \frac{1}{x} = \frac{\tan\theta}{6}$$

Now, $$9\left(x^2 - \frac{1}{x^2}\right) = 9\left(\frac{\sec^2\theta}{36} - \frac{\tan^2\theta}{36}\right)$$

$$= \frac{9}{36}(\sec^2\theta - \tan^2\theta) = \frac{1}{4}\times 1 = \frac{1}{4}$$

Question 71.

If 8r = cosec A and 8/x = cot A, find the value of $4(x^2-1/x^2)$

Solution:

Given $$8x = \text{cosec A and } \frac{8}{x} = \cot A$$

$$\Rightarrow \quad x = \frac{\text{cosec A}}{8} \text{ and } \frac{1}{x} = \frac{\cot A}{8}$$

Now, $$4\left(x^2 - \frac{1}{x^2}\right) = 4\left(\frac{\text{cosec}^2 A}{64} - \frac{\cot^2 A}{64}\right) = \frac{4}{64}(\text{cosec}^2 A - \cot^2 A) = \frac{1}{16}\times 1 = \frac{1}{16}$$

Question 72.

If 4x = sec θ and 4/x = tan θ, find the value of $8(x^2-1/x^2)$

Solution:

Given $$4x = \sec\theta \text{ and } \frac{4}{x} = \tan\theta$$

$$\Rightarrow \quad x = \frac{\sec\theta}{4} \text{ and } \frac{1}{x} = \frac{\tan\theta}{4}$$

Now, $$8\left(x^2 - \frac{1}{x^2}\right) = 8\left(\frac{\sec^2\theta}{16} - \frac{\tan^2\theta}{16}\right) = \frac{8}{16}(\sec^2\theta - \tan^2\theta) = \frac{1}{2}\times 1 = \frac{1}{2}$$

Short Answer Type Questions I [2 Marks]

Question 73.

Find the value of cosec 30° geometrically.

Solution:

For the value of cosec 30° consider an equilateral triangle ABC.

Let $2a$ be the length of each side

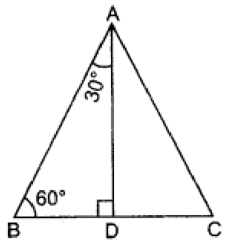
$\Rightarrow \quad AB = BC = CA = 2a$

Since each angle of equilateral triangle is 60°.

$\therefore \quad \Delta ABD \cong \Delta ACD$

$\therefore \quad BD = DC$ (CPCT)

and $\angle BAD = \angle CAD$



ΔABD is right angled triangle.

$$BD = \frac{1}{2}BC = a$$

Now, $$\text{cosec } 30° = \frac{AB}{BD} = \frac{2a}{a} = 2$$

Hence, the value of cosec 30° = 2.

Question 74.

Find the value of sec 60° geometrically

Solution:

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Consider an equilateral triangle ABC. Let $2a$ be the length of each side of the triangle.

$$AB = BC = CA = 2a$$

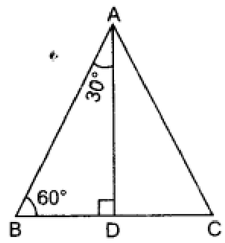
Since in an equilateral triangle each angle is 60°

$\therefore \quad \angle A = \angle B = \angle C = 60°$

Draw the perpendicular AD from A to BC

$\because \quad \Delta ABD \cong \Delta ACD$

$\therefore \quad \angle BAD = \angle CAD$ (CPCT)



ΔABD is right triangle, right angled at D

$\therefore \quad BD = \frac{1}{2}BC = a$

In ΔABD, $\sec 60° = \frac{AB}{BD}$

$$\sec 60° = \frac{2a}{a} = 2$$

Question 75.

Find the value of sec 45° geometrically

Solution:

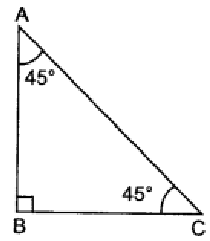
Consider ΔABC is an isosceles right angled Δ.

when $\angle B = 90°$ and $AB = BC = a$

$\therefore \quad AC = \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2}a$

$\angle A = \angle C = 45°$

$\therefore \quad \sec 45° = \frac{AC}{BC} = \frac{\sqrt{2}a}{a} = \sqrt{2} \Rightarrow \sec 45° = \sqrt{2}$



Short Answer Type Questions II [3 Marks]

Question 76.

Prove that: (cosec θ – sin θ). (sec θ – cos θ) =1/tan θ+cot θ

Solution:

LHS = (cosec θ – sin θ)(sec θ – cos θ) = (cosec θ – sin θ)(sec θ – cos θ)

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right) = \frac{(1-\sin^2\theta)(1-\cos^2\theta)}{\sin\theta\cos\theta} = \frac{\cos^2\theta\sin^2\theta}{\sin\theta\cos\theta}$$

$= \sin\theta\cos\theta$

$$RHS = \frac{1}{\tan\theta + \cot\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}} = \frac{1}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}} = \frac{\sin\theta\cos\theta}{\sin^2\theta + \cos^2\theta}$$

$= \sin\theta\cos\theta$

Hence, LHS = RHS

Question 77.

Prove that: (1 + cot A – cosec A) (1 + tan A + sec A) = 2.

Solution:

Taking LHS = (1 + cot A – cosec A)(1 + tan A + sec A)

$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right) = \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$

$$= \frac{(\sin A + \cos A)^2 - 1}{\sin A\cos A} = \frac{\sin^2 A + \cos^2 A + 2\sin A\cos A - 1}{\sin A\cos A}$$

$$= \frac{1 + 2\sin A\cos A - 1}{\sin A\cos A} = \frac{2\sin A\cos A}{\sin A\cos A} = 2$$

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Question 78.

Prove that: sin θ (1 + tan θ) + cos θ (1 + cot θ) = sec θ + cosec θ.

Solution:

$$\text{LHS} = \sin\theta(1+\tan\theta) + \cos\theta(1+\cot\theta)$$

$$= \frac{\sin\theta(\cos\theta+\sin\theta)}{\cos\theta} + \cos\theta\left(\frac{\sin\theta+\cos\theta}{\sin\theta}\right) = (\cos\theta+\sin\theta)\left[\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right]$$

$$= \frac{(\cos\theta+\sin\theta)}{\cos\theta\sin\theta}(\sin^2\theta+\cos^2\theta) = \frac{\cos\theta}{\cos\theta\sin\theta}+\frac{\sin\theta}{\cos\theta\sin\theta} = \text{cosec}\,\theta + \sec\theta$$

2009

Very Short Answer Type Questions [1 Mark]

Question 79.

If sin θ = 1/3, then find the value of (2 cot² θ + 2).

Solution:

$$2(\cot^2\theta+1) = 2\,.\,\text{cosec}^2\theta = \frac{2}{\sin^2\theta} = \frac{2}{(1/9)} = 18 \qquad \left(\because \sin^2\theta = \frac{1}{9}\right)$$

Question 80.

If sec2 θ(1 + sin θ) (1 – sin θ) = k, then find the value of k.

Solution:

$$\sec^2\theta\,(1+\sin\theta)\,(1-\sin\theta) = k$$

$$\Rightarrow \sec^2\theta\,(1-\sin^2\theta) = k \Rightarrow \sec^2\theta\,.\,\cos^2\theta = k \Rightarrow k = 1$$

Question 81.

If sec A = 15/7 and A + B = 90°, find the value of cosec B

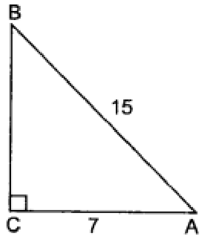
Solution:

Given $\sec A = \frac{15}{7}$

Also, $A + B = 90° \Rightarrow B = 90° - A$

$\Rightarrow \text{cosec}\,B = \text{cosec}\,(90 - A) = \sec A = \frac{15}{7}$

$\Rightarrow \text{cosec}\,B = \frac{15}{7}$



Short Answer Type Question I [2Marks]

Question 82.

Simplify $\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta} + \sin\theta\cos\theta$

Solution:

$$\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta} + \sin\theta\cos\theta$$

$$= \frac{(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta)}{(\sin\theta+\cos\theta)} + \sin\theta\cos\theta$$

$$= 1 - \sin\theta\cos\theta + \sin\theta\cos\theta = 1$$

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Question 83.

Without using trigonometric table evaluate:

$$\frac{\cos 58^\circ}{\sin 32^\circ}+\frac{\sin 22^\circ}{\cos 68^\circ}-\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ}\cot 72^\circ \cot 55^\circ$$

Solution:

$$\frac{\cos 58^\circ}{\sin 32^\circ}+\frac{\sin 22^\circ}{\cos 68^\circ}-\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ}\cot 72^\circ \cot 55^\circ$$

$$\frac{\cos 58^\circ}{\sin(90^\circ-58^\circ)}+\frac{\sin 22^\circ}{\cos(90^\circ-22^\circ)}-\frac{\cos 38^\circ \operatorname{cosec}(90^\circ-38^\circ)}{\tan 18^\circ \tan(90^\circ-55^\circ)\sqrt{3}}\cdot\cot(90^\circ-18^\circ)\cot 55^\circ$$

$$\frac{\cos 58^\circ}{\cos 58^\circ}+\frac{\sin 22^\circ}{\sin 22^\circ}-\frac{\cos 38^\circ \sec 38^\circ}{\tan 18^\circ \cot 55^\circ(\sqrt{3})}\cdot\tan 18^\circ \cot 55^\circ$$

$$1+1-\frac{1}{\sqrt{3}}=2-\frac{1}{\sqrt{3}}$$

Question 84.

Evaluate: $\frac{2}{3}\operatorname{cosec}^2 58^\circ-\frac{2}{3}\cot 58^\circ.\tan 32^\circ-\frac{5}{3}\tan 13^\circ.\tan 37^\circ.\tan 45^\circ.\tan 53^\circ.\tan 77^\circ$

Solution:

$$\frac{2}{3}\operatorname{cosec}^2 58^\circ-\frac{2}{3}\cot 58^\circ\tan 32^\circ-\frac{5}{3}\tan 13^\circ\tan 37^\circ\tan 45^\circ\tan 53^\circ\tan 77^\circ$$

$$=\frac{2}{3}\{\operatorname{cosec}^2 58^\circ-\cot 58^\circ\cot(90^\circ-32^\circ)\}-\frac{5}{3}\tan 13^\circ.\tan 37^\circ\times 1 \times\cot(90^\circ-53^\circ)\times\cot(90^\circ-77^\circ)$$

$$=\frac{2}{3}(\operatorname{cosec}^2 58^\circ-\cot 58^\circ.\cot 58^\circ)-\frac{5}{3}\tan 13^\circ.\tan 37^\circ.\cot 37^\circ.\cot 13^\circ$$

$$=\frac{2}{3}\times 1-\frac{5}{3}\tan 13^\circ.\tan 37^\circ\times\frac{1}{\tan 37^\circ}\times\frac{1}{\tan 13^\circ}$$

$$=\frac{2}{3}-\frac{5}{3}\times 1=-1$$

Question 85.

Prove that : $\sec^2\theta-\left[\frac{\sin^2\theta-2\sin^4\theta}{2\cos^4\theta-\cos^2\theta}\right]=1$

Solution:

$$\text{LHS}=\sec^2\theta-\frac{\sin^2\theta-2\sin^4\theta}{2\cos^4\theta-\cos^2\theta}$$

$$\sec^2\theta-\frac{\sin^2\theta(1-2\sin^2\theta)}{\cos^2\theta(2\cos^2\theta-1)}=\sec^2\theta-\frac{\sin^2\theta(\cos^2\theta-\sin^2\theta)}{\cos^2\theta(\cos^2\theta-\sin^2\theta)}$$

$$=\frac{1}{\cos^2\theta}-\frac{\sin^2\theta}{\cos^2\theta}=\frac{1-\sin^2\theta}{\cos^2\theta}=\frac{\cos^2\theta}{\cos^2\theta}=1=\text{RHS}$$

Hence proved.

Get More Learning Materials Here : CLICK HERE www.studentbro.in